**Linear programming Problem (LPP)** is an optimization technique for a system of linear constraints and a linear objective function. Where **objective function** defines the quantity to be optimized, and the goal of linear programming is to find the values of the variables that optimize (maximize or minimize) the objective function.

Types of linear programming : we will study in two type (A) Formulating the problem and (B) Solution of the problems (by graphical method only as per syllabus)

**(A) Formulating a problem : Let us consider the following example:**

**Example:** Consider a chocolate manufacturing company that produces only two types of chocolate – A and B. Both the chocolates require Milk and Choco only.  To manufacture each unit of A and B, the following quantities are required:

* Each unit of A requires 1 unit of Milk and 3 units of Choco
* Each unit of B requires 1 unit of Milk and 2 units of Choco

The company  has a total of 5 units of Milk and 12 units of Choco. On each sale, the company makes a profit of

* Rs 6 per unit A sold
* Rs 5 per unit B sold.

Now, the company wishes to maximize its profit. How many units of A and B should it produce respectively?

**Solution:** The first thing we have to do is represent the problem in a tabular form as below for better understanding.

|  |  |  |  |
| --- | --- | --- | --- |
| Type of chocolate | Milk | Choco | Profit per unit |
| A | 1 | 3 | Rs 6 |
| B | 1 | 2 | Rs 5 |
| Total | 5 | 12 |  |

Let the total number of units produced for A = X

Let the total number of units produced for B = Y

And , the total profit is represented by Z

The total profit the company is obtained by the total number of units of A and B produced multiplied by its price (per-unit profit) of Rs 6 and Rs 5 respectively.

**i.e Profit: Max Z = 6X+5Y**

which means we have to maximize Z.

The company will try to produce as many units of A and B to maximize the profit. But the resources Milk and Choco are available in a limited amount.

As per the above table, each unit of A and B requires 1 unit of Milk. The total amount of Milk available is 5 units. To represent this mathematically,

**X+Y ≤ 5**

Also, each unit of A and B requires 3 units & 2 units of Choco respectively. The total amount of Choco available is 12 units. To represent this mathematically,

**3X+2Y ≤ 12**

Also, the values for units of A can only be integers.

So we have two more constraints, **X ≥ 0  &  Y ≥ 0 (because production can not be -ve)**

For the company to make maximum profit, the above inequalities have to be satisfied.

**This is called formulating a real-world problem into a mathematical model.**

**Common terms used in Linear Programming**

Let us define some terminologies used in Linear Programming using the above example.

* **Decision Variables:**The decision variables are the variables that will decide the output. They represent the ultimate solution. To solve any problem, we first need to identify the decision variables. For the above example, the total number of units for A and B denoted by X & Y respectively are my decision variables.
* **Objective Function:**It is defined as the objective of making decisions. In the above example, the company wishes to increase the total profit represented by Z. So, profit is the objective function.
* **Constraints:**The constraints are the restrictions or limitations on the decision variables. They usually limit the value of the decision variables. In the above example, the limit on the availability of resources Milk and Choco are the constraints.
* **Non-negativity restriction:**For all linear programs, the decision variables should always take non-negative values. This means the values for decision variables should be greater than or equal to 0.

**The process of formulating a Linear Programming problem :**

To formulate a LPP we should follow the following steps:

1. Identify the decision variables (x,y,..)
2. Write the objective function (express in linear inequality)
3. Mention the constraints(express in linear inequality)
4. Explicitly state the non-negativity restriction.

For a problem to be a linear programming problem, the decision variables, objective function and constraints all have to be linear functions.

Example : A store has requested a manufacturer to produce pants and sports jackets.

For materials, the manufacturer has 750 m² of cotton textile and 1,000 m² of polyester. Every pair of pants (1 unit) needs 1 m² of cotton and 2 m² of polyester. Every jacket needs 1.5 m² of cotton and 1 m² of polyester.

The price of the pants is fixed at Rs 50 and 40.

What is the number of pants and jackets that the manufacturer must give to the stores so that these items obtain a maximum sale?

**1 Choose the decision variables (unknowns).**

**x = number of pants**

**y = number of jackets**

**2. Write the** [**objective function**](https://www.superprof.co.uk/resources/academic/maths/linear-algebra/linear-programming/linear-programming.html#of).

**f(x,y) = Z (max)= 50x + 40y**

**3 .** To write the constraints, use a table:

|  |  |  |  |
| --- | --- | --- | --- |
| Goods | Pants | Jackets | Profit per unit (price) |
| Cotton | 1 | 1.5 | 750 |
| polyster | 2 | 1 | 1000 |

x + 1.5y ≤ 750 or **2x+3y ≤ 1500 (multiplying by 2)**

**and 2x + y ≤ 1000**

As the number of pants and jackets are natural numbers, there are two more constraints:

**x ≥ 0**

**y ≥ 0**

this is the formulation of LPP

i.e Z=50X + 40Y

2X+3y **≤1500**

**2x + y ≤ 1000**

**x ≥ 0 , y ≥ 0**

**to be continue next class**