1

SEMESTER-II
MAT-RC-2016: Algebra

Total Marks: 100 (External: 80, Internal Assessment: 20)
Per week 5 Lectures, 1 Tutorial, Credits 6
Each unit carry equal credit

S.A.S

Unit 1: Theory of Equations and Expansions of Trigonometric Functions: Fundamental Theorem of Algebra, Relation between roots and coefficients of *n*th degree equation, Remainder and Factor Theorem, Solutions of cubic and biquadratic equations, when some conditions on roots of the equation are given, Symmetric functions of the roots for cubic and biquadratic; De Moivre's theorem (both integral and rational index), Solutions of equations using trigonometry and De Moivre's theorem, Expansion for in terms of powers of in terms of cosine and sine of multiples of x.
[2] Chapter 3, Chapter 4 [3] Chapter 7 (Sections 7.6, and 7.7)

P.C

Unit 2: Matrices: Types of matrices, Rank of a matrix, Invariance of rank under elementary transformations, Reduction to normal form, Solutions of linear homogeneous and nonhomogeneous equations with number of equations and unknowns up to four; Cayley-Hamilton theorem, Characteristic roots and vectors.
[4] Chapter 3 (Sections 3.2, 3.5, 3.7, 3.9) Chapter 2 (Sections 2.1 to 2.5)Chapter 7 (Section 7.1, and Example 7.2.2)

AST

Unit 3: Groups, Rings and Vector Spaces: Integers modulo n, Permutations, Groups, Subgroups, Lagrange's theorem, Euler's theorem, Symmetry Groups of a segment of a line, and regular n-gons for $n = 3$, 4, 5 and 6; Rings and subrings in the context of C[0,1] and Definition and examples of a vector space, Subspace and its properties, Linear independence, Basis and dimension of a vector space.
[1] Chapter 1 (Section 1.4), and Chapter 2 (Section 2.3) Chapter 3 (Sections 3.1, 3.2, 3.3 and 3.6), and Chapter 5 (Section 5.1) [4] Chapter 4 (Sections 4.1, 4.3 and 4.4)

Text Books:

1. Beachy, John A., & Blair, William D. (2006). *Abstract Algebra* (3rd ed.). Wavel and Press, Inc.
2. Burnside, William Snow (1979). *The Theory of Equations*, Vol. 1 (11th ed.) S. Chand & Co. Delhi. Fourth Indian Reprint.
3. Gilbert, William J., & Vanstone, Scott A. (1993). *Classical Algebra* (3rd ed.). Waterloo Mathematics Foundation, Canada.
4. Meyer, Carl D. (2000). *Matrix Analysis and Applied Linear Algebra*. Society for Industrial and Applied Mathematics (Siam).

Reference Books:

1. Dickson, Leonard Eugene (2009). *First Course in The Theory of Equations*. The Project Gutenberg EBook (http://www.gutenberg.org/ebooks/29785)
2. Gilbert, William J. (2004). *Modern Algebra with Applications* (2nd ed.). Wiley Interscience, John Wiley & Sons.

Part-B ②

Symmetrical functions of roots:

A function involving more than one vari... is said to be symmetrical if the functi... is not changed by interchanging any two of the variables.

Example: Symmetrical functions of the variables x and y are

i) $x+y$ 2) x^2+xy+y^2 iii) x^3+y^3+3xy etc.

Example: Symmetrical functions of three variables x, y and z are

i) $x^2+y^2+z^2$ ii) $x^3+y^3+z^3$, iii) $xy+yz+zx$.

Note Any Symmetrical function of the roots of an equation can be expressed in terms of the fundamental symmetrical functions of the roots of the equation.

Sign of $\Sum$:

① $\sum\alpha = \alpha+\beta+\gamma$

② $\sum\alpha\beta = \alpha\beta+\alpha\gamma+\beta\gamma$

3) $\sum\frac{1}{\alpha+\beta} = \frac{1}{\alpha+\beta}+\frac{1}{\beta+\gamma}+\frac{1}{\alpha+\gamma}$

4) $\sum(\alpha-\beta)^2 = (\alpha-\beta)^2+(\beta-\gamma)^2+(\alpha-\gamma)^2$,

Problems. (3)

1) If α, β, γ are the roots of the equation $x^3+px^2+qx+r=0$, find the value of $\alpha^2+\beta^2+\gamma^2$. Ans → p^2-2q.

2) If α, β, γ are the roots of the equation $ax^3+bx^2+cx+d=0$ find the value of $\alpha^3+\beta^3+\gamma^3$? Ans → $\frac{3abc-b^3-3a^2d}{a^3}$

3) If α, β, γ are the roots of the equation $ax^3+bx^2+cx+d=0$ find the value of $\sum\alpha^2\beta$ and $\alpha^3+\beta^3+\gamma^3$. Ans → $\frac{3ad-bc}{a^2}$, $\frac{3abc-b^3-3a^2d}{a^3}$

4) If α, β, γ are the roots of the equation $x^3+px+q=0$ find $\sum\frac{1}{\alpha+\beta}$. Ans → p/q.

92

5) If α, β, γ are the roots of the equation $x^3+px^2+qx+r=0$, find $\beta^2\gamma^2+\gamma^2\alpha^2+\alpha^2\beta^2$ and $\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}$.

6) If α, β, γ are the roots of the equation $x^3+px+q=0$, find $(\beta+\gamma-2\alpha)(\gamma+\alpha-2\beta)(\alpha+\beta-2\gamma)$

7) If α, β, γ are the roots of the equation $x^3+px^2+qx+r=0$ find $(\beta+\gamma-3\alpha)(\gamma+\alpha-3\beta)(\alpha+\beta-3\gamma)$

8) If α, β, γ, are the roots of $x^3+px^2+qx+r=0$ find $\left(\frac{1}{\beta}+\frac{1}{\gamma}-\frac{1}{\alpha}\right)\left(\frac{1}{\gamma}+\frac{1}{\alpha}-\frac{1}{\beta}\right)\left(\frac{1}{\alpha}+\frac{1}{\beta}-\frac{1}{\gamma}\right)$.

) If α, β, γ are the roots of $x^3+px^2+qx+r=0$ find $\sum\alpha^3\beta^3$.