Field

* Field:- A ring R with at least two elements is called a field if

1) R is commutative
11) R has unity
111) R is Such that each non zero element posses multiplicative inverse.

* Example:- The ring of rational number $(Q, +, \cdot)$ is a field.

soln ① The ring of rational number is a commutative ring.

11) The ring of rational number has unity.

111) The ring of rational number is Such that each non-zero element posses multiplication inverse.

Hence the system, the ring of $(Q, +, \cdot)$ is a field i.e the ring of rational number Q is a field.

* Some general properties of field:-

1) Every field is an integral domain.

2) A skew field has no divisor of zero.

3) A finite commutative ring without zero divisor is a field.

Every finite integral is a field.

* Note:- Integral domain:- A ring R is called an integral domain if
1) R is commutative.
11) R has unit element.
111) R is without zero divisor.

Unit-3 2nd Sem Group ① Previous Knowledge.

Group

Defⁿ:- A system $(G,\cdot)$, where G is a non-empty set and $\cdot$ is a binary operation on G, is called a group if it satisfies the following postulates:-

i) Closure property i.e $a\cdot b \in G \ \forall a,b \in G$

ii) Associative law i.e $(a\cdot b)\cdot c = a\cdot(b\cdot c) \ \forall a,b,c \in G$.

iii) Existance of identity i.e there exist an element $e \in G$ such that $a\cdot e = a = e\cdot a \ \forall a \in G$ (e = identity)

iv) Existance of inverse i.e for each $a \in G$, there exist an element $a^{-1} \in G$ such that $a\cdot a^{-1} = e = a^{-1}\cdot a$.

In addition to the above four postulates, if the following postulate is also satisfied, then the group is called a commutative group or an abelian group.

i.e (v) Commutative law: $a\cdot b = b\cdot a \ \forall a,b \in G$.

Note:- If the postulate (v) does not hold in the system $(G,\cdot)$ then the system is called a non-abelian group.

Note:- In the system $(G,\cdot)$ if the set G is finite then the system $(G,\cdot)$ is called a finite group. Otherwise the system $(G,\cdot)$ is called an infinite group.

Example:- Show that the set I of all integers i.e

$$I = \{\ldots, -4,-3,-2,-1,0,1,2,3,4,\ldots\}$$

is a group w.r.to the operation of addition of integers. Hence prove that the group is an abelian group.

Solⁿ: We can say the system $(I,+)$ be a group if we show that the system $(I,+)$ satisfies the four postulates of group.

i) Closure property: We know that the sum of two integers is also an integer. i.e $a+b \in I \ \forall a,b \in I$.

ii) Associative Law :- we know that the addition of integers is an composition i.e $(a+b)+c = a+(b+c)$ $\forall a,b,c \in I$

iii) Existance of identity :- The number $0 \in I$. Also we have $a+0 = a = 0+a$ $\forall a \in I$. Hence 0 is the identity.

iv) Existance of inverse :- If $a \in I$, then $-a \in I$. Also we have $a+(-a) = 0 = (-a)+a$. Thus every integer posses additive inverse.

Hence we can say that the system $(I,\cdot)$ is a group w.r.to addition. Hence I is a group.

v) Since addition of integers is a commutative composition, therefore $(I,+)$ is an abelian group.

Note :- Since I contains an infinite number of elements, therefore $(I,+)$ is an abelian group of infinite order.

* Some general properties of group :-

1) The identity in a group is unique.

2) The inverse of each element of a group is unique.

3) If the inverse of a is a^{-1} then the inverse of a^{-1} is a.

i.e $(a^{-1})^{-1} = a$

4) The inverse of the product of two elements of a group is the product of the inverse taken in reverse order. i.e

$$(ab)^{-1} = b^{-1} \cdot a^{-1}$$

5) Concellation Laws hold in a group.

6) If a, b are any two elements of a group then the equations $ax = b$ and $ya = b$ have unique Solⁿ in the group.

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Ring

* **Ring**:- A system $(R, +, \cdot)$ where R is a non empty set and $+, \cdot$ are two binary compositions on R, is called a Ring if it satisfies the following postulates:-

For any $a, b, c \in R$,

1) $a+(b+c) = (a+b)+c$ (i.e addition is associative)
2) $a+b = b+a$ (i.e addition is commutative)
3) There exist an element 0 in R such that $0+a = a \; \forall a \in R$
4) For each element a in R there exist an element $(-a)$ in R such that $a+(-a) = 0$
5) $a\cdot(b\cdot c) = (a\cdot b)\cdot c$ [i.e multiplication is associative]
6) $a\cdot(b+c) = a\cdot b + a\cdot c$ → left distributive law
   $(b+c)\cdot a = b\cdot a + c\cdot a$ → right distributive law
   [i.e multiplication is distributive]

where $+$ and $\cdot$ will be called addition and multiplication respectively.

Example:- The set I of all integers is a ring w.r.to two binary operations [addition and multiplication] of integers.

Soln. We can say that the system $(I, +, \cdot)$ be a ring if we show the system $(I, +, \cdot)$ satisfies the six properties of ring.

1) We know that the addition of integers is an composition i.e $a+(b+c) = (a+b)+c \; \forall a, b, c \in I$.

2) We have the addition of integers is commutative. i.e
$$a+b = b+a \; \forall a, b \in I$$

3) The ... , therefore $0+a = a \; \forall a \in I$.

4) If $a \in I$ then $-a \in I$, therefore $a+(-a) = 0 \; \forall a \in I$.

(5) The product of integers is an associative composition i.e $a\cdot(b\cdot c) = (a\cdot b)\cdot c \ \forall a,b,c \in I$

(6) Multiplication of integers is distributive w.r.to addition of integers. i.e $a\cdot(b+c) = a\cdot b + a\cdot c$ and $(b+c)\cdot a = b\cdot a + c\cdot a$.

Hence the system ~~forming~~ $(I,+,\cdot)$ is a ring w.r.to addition and multiplication of integers. Hence I is a ring.

* Some general properties of ring :-

1) $a\cdot 0 = 0\cdot a = 0, \ \forall a \in R$

2) $a\cdot(-b) = -(ab) = (-a)\cdot b, \ \forall a,b \in R$

3) $(-a)\cdot(-b) = ab, \ \forall a,b \in R$

4) $a(b-c) = ab - ac, \ \forall a,b,c \in R$

5) $(b-c)\cdot a = ba - ca, \ \forall a,b,c \in R$.

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SEMESTER-II
MAT-RC-2016: Algebra

Total Marks: 100 (External: 80, Internal Assessment: 20)
Per week 5 Lectures, 1 Tutorial, Credits 6
Each unit carry equal credit

PC **Unit 1: Theory of Equations and Expansions of Trigonometric Functions:**
Fundamental Theorem of Algebra, Relation between roots and coefficients of nth degree equation, Remainder and Factor Theorem, Solutions of cubic and biquadratic equations, when some conditions on roots of the equation are given, Symmetric functions of the roots for cubic and biquadratic; De Moivre's theorem (both integral and rational index), Solutions of equations using trigonometry and De Moivre's theorem, Expansion for in terms of powers of in terms of cosine and sine of multiples of x.
[2] Chapter 3, Chapter 4 [3] Chapter 7 (Sections 7.6, and 7.7)

AST **Unit 2: Matrices:** Types of matrices, Rank of a matrix, Invariance of rank under elementary transformations, Reduction to normal form, Solutions of linear homogeneous and nonhomogeneous equations with number of equations and unknowns up to four; Cayley-Hamilton theorem, Characteristic roots and vectors.
[4] Chapter 3 (Sections 3.2, 3.5, 3.7, 3.9) Chapter 2 (Sections 2.1 to 2.5)Chapter 7 (Section 7.1, and Example 7.2.2)

SAS **Unit 3: Groups, Rings and Vector Spaces:** Integers modulo n, Permutations, Groups, Subgroups, Lagrange's theorem, Euler's theorem, Symmetry Groups of a segment of a line, and regular n-gons for $n = 3$, 4, 5 and 6; Rings and subrings in the context of C[0,1] and Definition and examples of a vector space, Subspace and its properties, Linear independence, Basis and dimension of a vector space.
[1] Chapter 1 (Section 1.4), and Chapter 2 (Section 2.3) Chapter 3 (Sections 3.1, 3.2, 3.3 and 3.6), and Chapter 5 (Section 5.1) [4] Chapter 4 (Sections 4.1, 4.3 and 4.4)

Text Books:

1. Beachy, John A., & Blair, William D. (2006). *Abstract Algebra* (3rd ed.). Wavel and Press, Inc.
2. Burnside, William Snow (1979). *The Theory of Equations*, Vol. 1 (11th ed.) S. Chand & Co. Delhi. Fourth Indian Reprint.
3. Gilbert, William J., & Vanstone, Scott A. (1993). *Classical Algebra* (3rd ed.). Waterloo Mathematics Foundation, Canada.
4. Meyer, Carl D. (2000). *Matrix Analysis and Applied Linear Algebra*. Society for Industrial and Applied Mathematics (Siam).

Reference Books:

1. Dickson, Leonard Eugene (2009). *First Course in The Theory of Equations*. The Project Gutenberg EBook (http://www.gutenberg.org/ebooks/29785)
2. Gilbert, William J. (2004). *Modern Algebra with Applications* (2nd ed.). Wiley Interscience, John Wiley & Sons.