

SEMESTER-IV

MAT-RC-4016: Real Analysis

Total Marks: 100 (Theory 80, Internal Assessment 20)

Per week 5 Lectures, 1 Tutorial, Credits 6

Each unit carry equal credit

Unit 1: Order completeness of Real numbers, Open and closed sets, Limit of functions, Sequential criterion for limits, Algebra of limits, Properties of continuous functions, Uniform continuity.

[1] Chapter 2 (Sections 2.1, and 2.2, Sections 2.3, and 2.4) Chapter 11 (Section 11.1, Definition and Examples only)

Unit 2: Sequences, Convergent and Cauchy sequences, Subsequences, Limit superior and limit inferior of a bounded sequence, Monotonically increasing and decreasing sequences, Infinite series and their convergences, Positive term series, Comparison tests, Cauchy's nth root test, D'Alembert's ratio test, Raabe's test, Alternating series, Leibnitz test, Absolute and conditional convergence.

[1] Chapter 3, (Sections 3.1, and 3.2,3.3,3.4,3.5,3.7), Chapter 9 [Section 9.1(excluding grouping of series) Sections 9.2 (Statements of tests only), 9.3 (9.3.1, and 9.3.2) Chapter 4 (Sections 4.1 to 4.3).Chapter 5 (Sections 5.1, 5.3 and 5.4 excluding continuous extension and approximation)

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Mathematics
Semester-IV
MAT-RE-4016
Real Analysis

Unit-1

Order Completeness of Real Numbers :-

Completeness implies that there are not any "gaps" or "Missing Point" in the real number line. But in case of rational numbers, whose corresponding number line has a "gap" at each irrational value.

Completeness :-

In the mathematical area of order theory, completeness properties assert the existence of certain infima or suprema of a given partially ordered set. The most familiar example is the completeness of the real numbers. A special use of the term refers to complete partial orders or complete lattices.

The motivation for considering completeness properties derives from the great importance of suprema (least upper bounds, joins " \vee ") and infima (greatest lower bounds, meets " \wedge ") to the theory of partial orders. The knowledge that certain types of subsets are guaranteed to

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have suprema or infima enables us to consider the computation of these elements as total operations on a partially ordered set. For this reason, partially ordered set with certain completeness properties can often be described as algebraic structures of a certain kind. So completeness axiom guarantees that for any non-empty subset of R that is bounded above has a least upper bound.

Types of Completeness properties :-

All completeness properties are described along a similar scheme: one describes a certain class of subsets of a partially ordered set that are required to have a supremum or required to have an infimum. Hence every completeness property has its dual, obtained by inverting ordered-dependent definitions in the given statement. Some of the notions are usually not dualized while others may be self-dual.

Finite Completeness :-

Simple completeness conditions arise from the consideration of all non-empty finite sets. An order in which all non-empty finite sets have both a supremum and a infimum is called a lattice. It suffices to require that all suprema and infima of two elements exist to obtain all non-empty finite ones; a straight forward induction argument shows that every finite non-empty supremum/infimum can be decomposed into a finite number of suprema/infima.