

Number System

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Introduction:

Arithmetic is the science that treats of numbers and of the methods of computing by means of them. A number expresses how many times a unit is taken. A unit denotes a single thing as one man, one metre, one rupee etc.

It is known that in Hindu-Arabic System we use ten symbols, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 called digits to represent any no. Expressing a no. in words is called numeration and representing a number in figures is called notation.

Numerals and Types of Numbers

We know that the nos. are expressed by means of figures - 1, 2, 3, 4, 5, 6, 7, 8, 9 and 0 - called digits. 0 is called insignificant digit whereas the others are called significant digits. A group of figures representing a number is called a numeral.

Types of Numbers

a) Natural Number
Numbers which we use for counting the objects are known as natural no. They are denoted by 'N'.
$$N = \{1, 2, 3, 4, 5, \dots\}$$

b) Whole Number: When we include zero in the natural nos., it is known as whole nos. They are denoted by 'W'.

$$W = \{0, 1, 2, 3, 4, 5, \dots\}$$

c) Prime Number :- A no. other than 1 is called a prime no. if it is divisible only by 1 and itself.

ex. Is 349 a prime no.?

Ans. yes, 349 is a prime no.

The sq. root of 349 is app. 19.

The prime no.s less than 19 are 2, 3, 5, 7, 11, 13, 17
clearly 349 is not divisible by any of them.

ex. Is 979 a prime no.?

Ans.:- No. The app. sq. root of 979 is 32.

Prime no.s less than 32 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31.

979 is divisible by 11. So it is not a prime no.

d) Composite Numbers :-

A no. other than 1, which is not a prime number is called a composite no.

e.g. 4, 6, 8, 9, 12, 14

Note :- 1 is neither prime nor composite.

e) Even no. :- The no. which is divisible by 2 is known as an even no.

e.g. 2, 4, 8, 10, 12, 14, ...

f) Odd No. :- The no. which is not divisible by 2 is known as an odd no.

e.g. 3, 5, 7, 9, 11, 13, 15, 17, 19, ...

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Consecutive Numbers:-

A series of no.s in which each is greater than that which precedes it by 1 is called consecutive numbers.

e.g. - 6, 7, 8.

2) Integers :- The set of no.s which consists of whole no.s and negative no.s is known as integers. It is denoted by I or Z.

e.g. $I = \{-4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \dots\}$

i) Rational No.s :- The no.s which can be written in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$ is called a rational no. It is denoted by Q.

When the no.s are written in fractions, they are known as rational no.s. It is denoted by Q.

e.g. $\frac{1}{2}, \frac{3}{4}, \frac{8}{9}$ are rational no.s

j) Irrational Numbers :- A no. which is not rational i.e. which cannot be expressed in the form $\frac{p}{q}$, p, q are integers and $q \neq 0$ is called an irrational no.

e.g. $\sqrt{3}, \pi, \sqrt{5}$.

k) Real Numbers :- A no. which is either rational or irrational is called a real no. It is denoted by R.

e.g. $\frac{2}{3}$ being a rational no. is a real no.
 $\sqrt{2}$ is an irrational no. is a real no.

Q. Which of the following is a rational no.?

$$(\sqrt{2})^2, 2\sqrt{2}, 2+\sqrt{2}, \frac{\sqrt{2}}{2}$$

Soln: $(\sqrt{2})^2 = 2^{\frac{1}{2} \times 2} = 2$ is a rational no.

Q. How many even no.'s are prime?

Ans: One and it is 2.

H.W. Q. Write the prime no.'s between 50 to 110.

Q. Which of the following no.'s are prime
97, 212, 137.

Soln: 97 has only multiplication factor

$$97 = 1 \times 97$$

\therefore 97 is a prime no.

$$212 = 1 \times 212$$

$$= 2 \times 106$$

$$= 4 \times 53 \text{ and so on.}$$

\therefore 212 is not a prime no. since it has more than two factors.

$$137 = 1 \times 137$$

\therefore 137 is a prime no.

Q. Which one of the following is always odd?

i) Sum of two odd no's.

ii) Difference of two odd no's.

iii) Product of two odd no's.

iv) None of these.

Ans: iii)

Note Sum of two even no's is always even

Ex Which of the following no.'s are prime numbers? (3)
i) 179 ii) 117 iii) 139

Soln: i) Given no. = 179.

Prime no's less than 13 are
2, 3, 5, 7, 11.

179 is not completely divisible by
any of the no's 2, 3, 5, 7, 11 and also 13.

∴ It is a prime no.

Sq. root by division method

$$\begin{array}{r} 13 \\ \hline 1 \overline{) 179} \\ \underline{1} \\ 23 \\ \underline{23} \\ 0 \\ \hline \end{array}$$

∴ App. sq. root
of 179 = 13.

ii) Given no. = 117.

It is not a prime no.

Since 117 is divisible by 13.

iii) Given no. = 139

We find that 139 is not completely
divisible by any of the no's

2, 3, 5, 7, 11. ~~and 13.~~

∴ 139 is a prime no.

Square root of 139 by division method:

$$\begin{array}{r} 11 \\ \hline 1 \overline{) 139} \\ \underline{1} \\ 29 \\ \underline{22} \\ 18 \\ \hline \end{array}$$

∴ App. sq. root of
139 = 11.

Division Algorithm

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Division Algorithm Formula

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder.}$$

- i) Dividend :- The no. which is to be divided is called dividend.
- ii) Divisor :- The no. by which the dividend is divided is called the divisor.
- iii) Quotient :- The number of times the divisor is contained in the dividend is called the quotient.
- iv) Remainder :- The no. which is left over after division is called the remainder.

Division Algorithm:

$$\begin{array}{r} \text{Divisor} \leftarrow 8 \quad \left. \begin{array}{l} 13 \rightarrow \text{Quotient} \\ 105 \rightarrow \text{Dividend} \\ - 8 \\ \hline 25 \\ 24 \\ \hline 1 \rightarrow \text{Remainder.} \end{array} \right\} \end{array}$$

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

$$105 = 8 \times 13 + 1.$$

Worked out Examples

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Ex-1 In a division sum, the quotient is 12 times and the divisor is 25 times the remainder and if the quotient is 144, find the dividend.

Soln: Given, quotient = 144.

A/q, Quotient = 12 times the remainder
 $\Rightarrow 144 = 12 \text{ times the remainder}$

$$\Rightarrow \text{remainder} = \frac{144}{12} = 12.$$

$$\begin{aligned} \text{Divisor} &= 25 \text{ times the remainder} \\ &= 25 \times 12 = 300. \end{aligned}$$

$$\begin{aligned} \therefore \text{Dividend} &= \text{Divisor} \times \text{Quotient} + \text{Remainder} \\ &= 300 \times 144 + 12 = 43212. \end{aligned}$$

Ex-2: Find the divisor when the quotient, dividend and the remainder are respectively 547, 171282 and 71.

Soln: Dividend = 171282

Quotient = 547.

Remainder = 71.

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

$$\Rightarrow 171282 = \text{Divisor} \times 547 + 71$$

$$\Rightarrow 171282 - 71 = \text{Divisor} \times 547$$

$$\Rightarrow \text{Divisor} = \frac{171211}{547}$$

$$\therefore \text{Divisor} = 313$$

$$\begin{array}{r} 171211 \quad | \quad 313 \\ \underline{1641} \\ 711 \\ \underline{547} \\ 1641 \\ \underline{1641} \\ \hline \end{array}$$

Divisibility Test

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Factors: In the expression $5 \times 8 = 40$, 5 and 8 are called factors and 40 is the product.

Factors of 40 are:

$$\begin{aligned} 40 &= 1 \times 40 \\ &= 2 \times 20 \\ &= 4 \times 10 \\ &= 5 \times 8 \end{aligned}$$

\therefore 1, 2, 4, 5, 8, 10, 20 are factors of 40.

Thus a given no. which divides a given no. exactly is called a factor of the given no.

e.g. $2 = 1 \times 2$, 1 and 2 are the factors of 2.

- Note:
- i) 1 is a factor of every no.
 - ii) Any no. is a factor of itself.
 - iii) A factor of a no. is always equal to or less than the no.

Ex-1 Find whether 121 is a factor of 1573.

Soln: We divide 1573 by 121 to obtain the remainder.

$$\text{Remainder} = 0.$$

\therefore 121 is a factor of 1573.

Ex. Determine whether 27 is a factor of 6561.

Soln: We divide 6561 by 27 and see whether the remainder is zero.

$$\begin{array}{r} 27 \overline{) 6561} \quad (243 \\ \underline{54} \\ 116 \\ \underline{108} \\ 81 \\ \underline{81} \\ 0 \end{array}$$

Since the remainder is zero, \therefore 27 is a factor of 6561.

Tests of Divisibility of No.'s

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- i) Divisibility by 2: A no. is divisible by 2 if its units digit is 0, 2, 4, 6 or 8
or
if the last digit is 0, 2, 4, 6 and 8.
e.g. 50, 162, 536.
- ii) Divisibility by 3: If the sum of its digits is divisible by 3. e.g. 378
- iii) Divisibility by 4: If the no. formed by its last two digits (units and tens) is divisible by 4. e.g. 824.
397542 is not divisible by 4 since 42 is not divisible by 4.
- iv) Divisibility by 5: If the digit in its units place is either 5 or 0 (if the last digit is 5 or 0). e.g. 78675.
- v) Divisibility by 6: If the no. is divisible by both 2 and 3.
e.g. 216 is divisible by 6 since it is divisible by 2 and 3.
- vi) Divisibility by 7: The difference betⁿ twice the unit digit of the given no. and the remaining part of the given no. should be multiple of 7 or it should be equal to 0.
e.g.
$$\begin{aligned} & 798 \\ &= 79 - 2 \times 8 \\ &= 63. \quad 63 \text{ is divisible by } 7. \end{aligned}$$

∴ 798 is divisible by 7.

Divisibility by 8: If the no. formed by the last three digit (units, tens and hundred's) is divisible by 8.

e.g. 568128 is divisible by 8 since 128 is divisible by 8.

e.g. 2954100 not divisible by 8.

Divisibility by 9: If the sum of its digits is a multiple of 9 (i.e. divisible by 9).

e.g. 567891; $5+6+7+8+9+1=36$ is divisible by 9.

Divisibility by 10: If the last digit of the no. is zero.

Divisibility by 11: If the difference of sum of the digits at the alternate places is divisible by 11.

e.g. $132 = (1+2) - 3 = 0$.

$10648 = (1+6+8) - (0+4) = 11$.

Ex. Is 284907205 divisible by 11?

Divisibility by 12: If the no. is divisible both by 3 and 4.

e.g. 144 is divisible both by 3 and 4.
So it is divisible by 12.

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Divisibility by 15: The no. should be divisible by both 3 and 5.

e.g. 97980 is divisible by 15

33125 is not divisible by 15.

Divisibility by 17: If the no. of ten's added to 12 times the no. of unit's is divisible by 17.

e.g. 153.

No. of ten's = 15

The reqd. sum = $15 + 12 \times 3 = 51$ which is divisible by 17.

\therefore 153 is divisible by 17.

Divisibility by 18: The no. should be divisible by both 2 and 9.

59850 \rightarrow divisible by 18.

58779 \rightarrow is not divisible by 18.