Definition of Logarithm:-

If for a positive real number a, $a \neq 1$

$$a^m = b$$

We say that m is the logarithm of b to the base a.

We write this as

$$\log_a b = m.$$

"log" being the abbreviation of the word "logarithm".

Examples:

$\because 2^3 = 8 \quad \therefore \log_2 8 = 3.$

$\because 3^2 = 9 \quad \therefore \log_3 9 = 2.$

$\because 5^3 = 125 \quad \therefore \log_5 125 = 3.$

$\because 7^0 = 1 \quad \therefore \log_7 1 = 0.$

$\because 9^{1/2} = 3 \quad \therefore \log_9 3 = \frac{1}{2}$

$\because 16^{\frac{1}{4}} = 2 \quad \therefore \log_{16} 2 = \frac{1}{4}.$

Ex Write the following in the form of logarithms:

i) $5^0 = 1$

$\because a^m = b \Rightarrow \log_a b = m.$

$5^0 = 1 \Rightarrow \log_5 1 = 0.$

ii) $\sqrt{9} = 3 \Rightarrow 9^{\frac{1}{2}} = 3 \Rightarrow \log_9 3 = \frac{1}{2}.$

iii) $7^3 = 343 \Rightarrow \log_7 343 = 3.$

Ex-2 Write the following in the form of logarithms.

i) $5^{-3} = \frac{1}{125}$

Soln: $\because a^m = b \therefore \log_a b = m$.

$5^{-3} = \frac{1}{125} \Rightarrow \log_5\left(\frac{1}{125}\right) = -3$.

ii) $10^{-5} = .00001 \Rightarrow \log_{10} .00001 = -5$

iii) $9^{-\frac{5}{2}} = \frac{1}{243} \Rightarrow \log_9\left(\frac{1}{243}\right) = -\frac{5}{2}$.

Ex-3 Write the following in the exponential form:

i) $\log_2 64 = 6$.

Soln: $\because \log_a b = m \Rightarrow a^m = b$.

$\therefore \log_2 64 = 6 \Rightarrow 2^6 = 64$.

ii) $\log_4 128 = \frac{7}{2} \Rightarrow 4^{\frac{7}{2}} = 128$.

iii) $\log_2\left(\frac{1}{8}\right) = -3 \Rightarrow 2^{-3} = \frac{1}{8}$

iv) $\log_{10} .01 = -2 \Rightarrow (10)^{-2} = .01$.

v) $\log_{10}(.001) = -3 \Rightarrow (10)^{-3} = .001$.

vi) $\log_6\left(\frac{1}{216}\right) = -3 \Rightarrow 6^{-3} = \frac{1}{216}$.

Find the value of each of the following:

i) $\log_2 32$

Soln: If $\log_a b = m$ Then $a^m = b$.

Let $\log_2 32 = m$.

Then $2^m = 32$.

$\Rightarrow 2^m = 2^5$

$\Rightarrow m = 5$.

$\Rightarrow \log_2 32 = 5$.

ii) $\log_8 4$

Soln: Let $\log_8 4 = m$

$\Rightarrow 8^m = 4$

$\Rightarrow (2^3)^m = 2^2 \Rightarrow 2^{3m} = 2^2$

$\Rightarrow 3m = 2 \Rightarrow m = \frac{2}{3}$.

$\therefore \log_8 4 = \frac{2}{3}$.

iii) $\log_2 \left(\frac{1}{8}\right)$.

Soln: Let $\log_2 \left(\frac{1}{8}\right) = m$

$\Rightarrow 2^m = \frac{1}{8}$

$\Rightarrow 2^m = \frac{1}{2^3} \Rightarrow 2^m = 2^{-3} \Rightarrow m = -3$.

$\therefore \log_2 \left(\frac{1}{8}\right) = -3$.

iv) $\log_{10} .01$.

Soln: Let $\log_{10} .01 = m$

$\Rightarrow 10^m = .01$

$\Rightarrow 10^m = \frac{1}{100} \Rightarrow 10^m = \frac{1}{10^2} \Rightarrow 10^m = 10^{-2}$

$\Rightarrow m = -2$. $\therefore \log_{10} .01 = -2$.

Laws of Logarithms :-

1. **First Law (Product formula):-**

$$\log_a (mn) = \log_a m + \log_a n \ , \quad a \text{ is real number } a>0,\ a\neq 1.$$

i.e. The logarithm of the product of two numbers is equal to the sum of their logarithms.

Pf :- Let $\log_a m = x$ and $\log_a n = y$.

Then $a^x = m$ and $a^y = n$.

Hence, $mn = a^x . a^y = a^{x+y}$

$\Rightarrow \log_a (mn) = x+y$ (by defn of logarithm).

$\Rightarrow \log_a (mn) = \log_a m + \log_a n$.

2. **Second Law (Quotient formula):-**

$$\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n \ , \quad a \text{ is a real number, } a>0,\ a\neq 1.$$

i.e. the logarithm of the quotient of two numbers is equal to the difference of their logarithms.

Pf :- Let $\log_a m = x$ and $\log_a n = y$.

Then $a^x = m$ and $a^y = n$

Hence $\dfrac{m}{n} = \dfrac{a^x}{a^y} = a^{x-y}$

$\Rightarrow \log_a\left(\frac{m}{n}\right) = x-y$. [by defn of logarithm]

$\Rightarrow \log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$.

3. Third Law (Power Formula):-

$\log_a m^n = n \log_a m$, a is a real number $a>0$, $a \neq 1$.

i.e. The logarithm of a number raised to a power is equal to the product of the index of the power multiplied by the logarithm of the number.

Pf:- Let $\log_a m = x$.

Then $a^x = m$.

Hence $m^n = (a^x)^n = a^{nx}$

$\Rightarrow a^{nx} = m^n$

$\Rightarrow \log_a (m^n) = nx$ [by def^n of logarithm]

$\Rightarrow \log_a (m^n) = n \log_a m$.

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Common Logarithms :-

Logarithms to the base 10 are called common Logarithms. When no base is mentioned, it should be taken as 10.

$\because \; 10^0 = 1 \quad \therefore \; \log_{10} 1 = 0$

$\because \; 10^1 = 10 \quad \therefore \; \log_{10} 10 = 1$

$\because \; 10^2 = 100 \quad \therefore \; \log_{10} 100 = 2$

$\because \; 10^4 = 10000 \quad \therefore \; \log_{10} 10000 = 4$

$\because \; 10^{-2} = \cdot 01 \quad \therefore \; \log_{10} 0{\cdot}01 = -2$

$\because \; 10^{-3} = \cdot 001 \quad \therefore \; \log_{10} 0{\cdot}001 = -3.$

Antilogarithm :-

If $\log N = t$, Then we say that

$N = \text{antilog } t.$

Thus to find N when $\log N$ (i.e t) is given, we find antilog t from ready-made antilog tables. This gives N.

Steps in finding Antilog n

In order to find the antilogarithm of a number, we use the decimal part of the number and read the antilog table exactly in a similar manner which is adopted for reading the log table. After finding the corresponding number from antilog table, we insert the decimal point according to following rules:

i) If the characteristic is n, Then the decimal point is inserted after $(n+1)$ digits.

ii) If the characteristic is n, the decimal point is inserted in such a way that the first significant figure is at the n^{th} place.

Examples

Number	Antilog
0·0629	1·161
1·0629	11·61
2·0629	116·1
3·0629	1161
4·0629	11610
$\bar{1}$·0629	·1161
$\bar{2}$·0629	·01161
$\bar{3}$·0629	·001161.