Laws of Exponents:-

I. Integral Exponents of a Real Number:-

i) **Positive Integral Power:-** For any real number 'a' and a ~~the~~ Positive integer 'n', we define a^n as

$$a^n = a \times a \times a \times \cdots \times a \quad (n \text{ factors}).$$

a^n is called the n^{th} power of a.

The real number 'a' is called the base and 'n' is called the exponent of the n^{th} power of 'a'.

Examples

i) $2^3 = 2 \times 2 \times 2 = 8$.

ii) $\left(\frac{3}{2}\right)^3 = \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} = \frac{27}{8}$.

iii) $\left(\frac{-3}{4}\right)^2 = \frac{-3}{4} \times \frac{-3}{4} = \frac{9}{16}$.

Note:- For any non-zero number 'a' we define $a^0 = 1$.

Thus $3^0 = 1$, $5^0 = 1$, $\left(\frac{3}{4}\right)^0 = 1$.

ii) **Negative Integral power:-** For any non-zero real number 'a' and a positive integer 'n', we define

$$a^{-n} = \frac{1}{a^n}.$$

Thus i) $5^{-3} = \frac{1}{5^3} = \frac{1}{5 \times 5 \times 5} = \frac{1}{125}$.

ii) $\left(\frac{3}{2}\right)^{-3} = \frac{1}{\left(\frac{3}{2}\right)^3} = \frac{1}{\frac{3}{2} \times \frac{3}{2} \times \frac{3}{2}} = \frac{8}{27}$.

Indices:

Indices are numeric values that act as Power or exponent to a particular number.

In other words, Indices are the power or exponent of a value.

Laws of Indices:

i) $a^m \times a^n = a^{m+n}$

ii) $a^m \div a^n = a^{m-n}$

iii) $(a^m)^n = a^{mn}$

iv) $a^{\frac{1}{m}} = \sqrt[m]{a}$

v) $a^{-m} = \frac{1}{a^m}$

vi) $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

vii) $a^0 = 1.$

viii) $(a^m)^n = a^{mn}$

ix) $(ab)^n = a^n b^n$

x) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Example Express the following in index form:-

$\sqrt{15} = (15)^{\frac{1}{2}}$

$\sqrt{6x^7} = (6x^7)^{\frac{1}{2}} = 6^{\frac{1}{2}} \times x^{\frac{7}{2}}$

$2\sqrt[4]{a^5} = 2a^{\frac{5}{4}}$

$\sqrt[n]{a^m} = a^{\frac{m}{n}}.$

Worked Out Examples

Ex Evaluate each of the following:

i) $\left(\frac{2}{11}\right)^4 \times \left(\frac{11}{3}\right)^2 \times \left(\frac{3}{2}\right)^3$

$= \frac{2^4}{11^4} \times \frac{11^2}{3^2} \times \frac{3^3}{2^3}$

$= \frac{2 \times 3}{11^2} = \frac{6}{121}$

ii) $\left(\frac{1}{2}\right)^5 \times \left(\frac{-2}{3}\right)^4 \times \left(\frac{3}{5}\right)^{-1}$

$= \left(\frac{1}{2}\right)^5 \times \left(-\frac{2}{3}\right)^4 \times \frac{1}{\frac{3}{5}}$

$= \frac{1^5}{2^5} \times \frac{(-2)^4}{3^4} \times \frac{5}{3} = \frac{1^5 \times (-2)^4 \times 5}{2^5 \times 3^4 \times 3}$

$= \frac{1 \times 16 \times 5}{32 \times 81 \times 3} = \frac{5}{486}$

iii) $2^{55} \times 2^{60} - 2^{97} \times 2^{18}$

$= 2^{55+60} - 2^{97+18}$

$= 2^{115} - 2^{115} = 0$

iv) $\left(\frac{2}{3}\right)^2 \times \left(\frac{2}{5}\right)^{-3} \times \left(\frac{3}{5}\right)^2$

$= \frac{2^2}{3^2} \times \frac{1}{\left(\frac{2}{5}\right)^3} \times \frac{3^2}{5^2}$

$= \frac{2^2}{3^2} \times \frac{1}{\frac{2^3}{5^3}} \times \frac{3^2}{5^2} = \frac{2^2 \times 5^3 \times 3^2}{3^2 \times 2^3 \times 5^2}$

$= \frac{5}{2}$ #